BU CS 332 – Theory of Computation

Lecture 9:

- Midterm I review

Reading:

Sipser Ch 0-2.3

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Midterm I Topics

Deterministic FAs (1.1)

$\{x \mid \ldots\}$

- Given an English description of a language L, write its formal description (and vice versa)
- Given an English or formal description of a language L, draw the state diagram of a DFA recognizing L (and vice versa)
- Know the formal definition of a DFA (A DFA is a 5 tuple…) and convert between state diagram and formal description
- Know the formal definition of how a DFA computes
- Regular operations: Union, concatenation, star and closure of regular languages under regular operations, construction for closure under complement
  - Cross-product construction for union/intersection

Nondeterministic FAs (1.2)

- Given an English or formal description of a language L, draw the state diagram of an NFA recognizing L (and vice versa)
- Know the formal definition of an NFA
- Know the power set construction for converting an NFA to a DFA
- Proving closure properties: Know the constructions for union, concatenation, star
- Recall other closure properties: reverse, intersection, complement

Regular Expressions (1.3)

- Given an English or formal description of a language L, construct a regex generating L (and vice versa)
- Formal definition of a regex
- Know how to convert a regex to an NFA
- Know how to convert a DFA/NFA to a regex

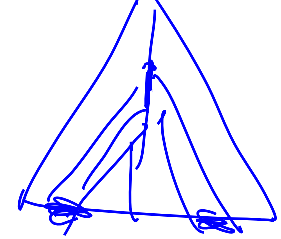
Non-regular Languages (1.4)

- Know the proof ideas for the pumping lemma for regular languages
- Understand the statement of the pumping lemma and how to apply it
- Beyond the pumping lemma: Showing languages are non-regular by combining pumping lemma with closure properties

Context-free Grammars (2.1)

- Given an English or formal description of a language L, give a CFG ~~(in Backus-Naur form)~~ generating L (and vice versa)
- Formal definition of a CFG (A CFG is a 4-tuple…), context-free languages
- Parse trees, derivations

Non-context-free Languages



- Know the proof ideas for the pumping lemma for CFLs
- Understand the statement of the pumping lemma and how to apply it
- Beyond the pumping lemma: Showing languages are non-context-free by combining pumping lemma with closure properties

- closure properties for CFLs
  - union
  - concatenation

Exam Tips

Study Tips

- Review problems from HW 0-3, discussion sections 1-3, solved exercises/problems in Sipser, and suggested exercises on the homework
  - We will literally ask you a question from the homework exercises, so make sure you understand these

- While you are not required to prepare a cheat-sheet, this is a great way to study!

Study Tips

- Make sure you know how to solve the problems on the practice midterm and are familiar with the format. The format/length of the real midterm will be very similar.

- If you need more practice, there are lots of problems in the book. We're happy to talk about any of these problems in office hours.

For the exam itself

- You may cite without proof any result…
  - Stated in lecture
  - Stated and proved in the main body of the text (Ch. 0-2.3)
  - These include worked-out examples of state diagrams, regexes, CFGs, non-regular/non-CF languages

- Not included above: homework problems, discussion problems, (solved) exercise/problems in the text

- Showing your work / explaining your answers will help us give you partial credit

Practice Problems

Let L be $L(G)$ where G is of the form: (V, Σ, R, S)

Every rule in R is of the form

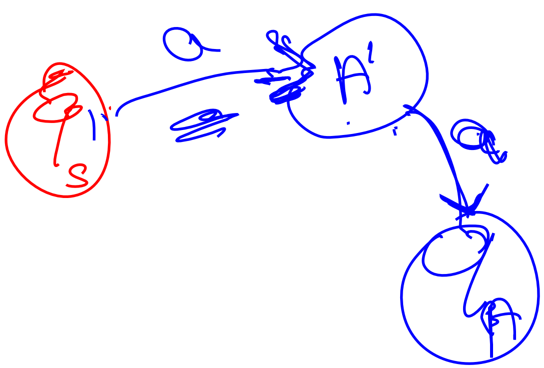
$$A \rightarrow a A'$$
$$A \rightarrow a$$

$a \in \Sigma \cup \varepsilon$

$A' \in V$

Show that L is regular.

Answer: build an NFA for L.



$(S_A, q_s, \{q_f\}, \Sigma, \delta)$

$\{q_f\} \cup V \quad S \quad \{q_f\} \quad \Sigma$

$\delta = \{ A \xrightarrow{a} A' \mid A \rightarrow aA'$

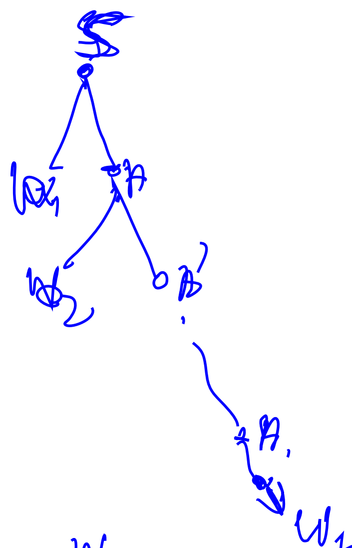
Proof of construction

Claim: $L(G) = L(N)$

- Assume $w \in L(G)$.

  Show: $w \in L(N)$

If $w \in L(G)$ then consider the parse tree of w:

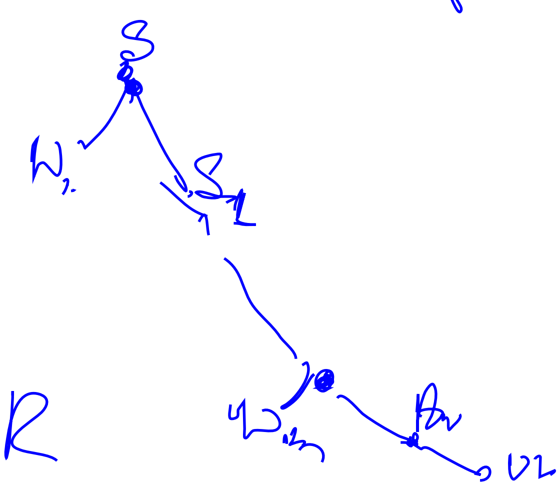


$w = w_1 \ldots w_n$

Then by construction there exists a path in N from

q_s to q_A when the label of the i-th edge in the path is w_i.

Assume $w \in L(N)$
Show $w \in L(G)$
$w \in L(N) \Rightarrow \exists$ path in N from q_s to q_A
where the label of ith edge in path is w_i
$\Rightarrow$ create a parse tree for w in G:



we know by construction that for all i

$(A_{i-1} \to w_i A_i) \in R$

→ to do: show that for any regular L $\exists$ G of the form $R = \{A \to aA'\}$ s.t. $L = L(G)$

Regular Languages

Name six operations under which the regular languages are closed

Prove or disprove: The non-regular languages are closed under union

$L = \{0^n 1^n\}$

$\overline{L} = \Sigma^* - \{0^n 1^n\}$

Disprove

counter example:

Let L be a non-regular Language

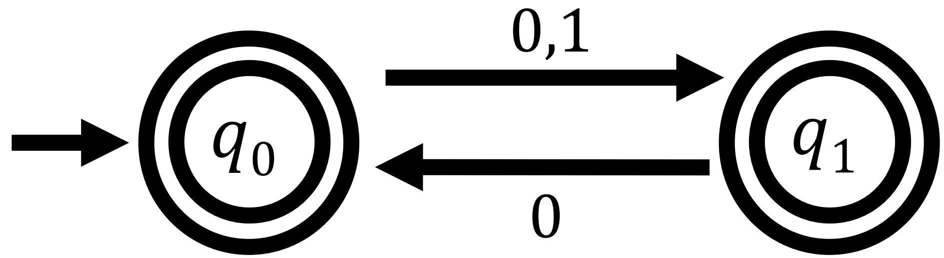
$\Rightarrow \overline{L}$ is not regular

(by closure of regular languages under complement)

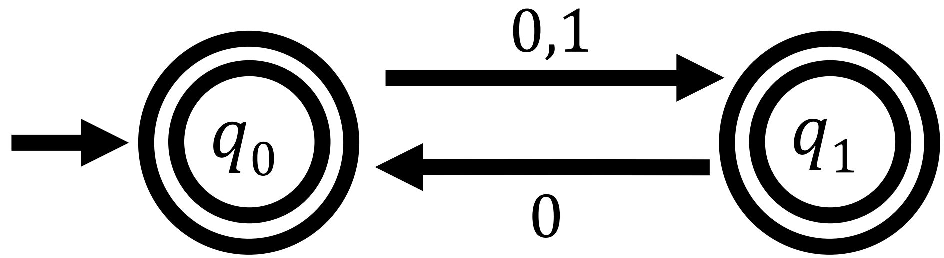
however, $L \cup \overline{L} = \Sigma^*$ is regular.

Give the state diagram of an NFA recognizing the language (01 U 10)*

Give an equivalent regular expression for the following NFA



Give an equivalent regular expression for the following NFA



Let R be a regular expression with n symbols. If we convert R into an NFA using the procedure described in class, how many states could it have in the worst case?

Is the following language regular?

$$\{a^n a^n | n \geq 0\}$$

Is the following language regular?

$$\{0^n 1^n \mid 0 \leq n \leq 2020\}$$

Let $L = \{w \in \{0,1\}^* \mid w \text{ has the same number of 0s and 1s}\}$.
Let p be a pumping length and $s = (01)^p$.
Give a decomposition of $s = xyz$ which **can** be pumped in L.
Is L regular?

Context-Free Languages

Name three operations under which the context-free languages are closed.

Name two operations under which the CFLs are *not* closed

What language is generated by the CFG

$$S \to aSb \mid bY \mid Ya$$
$$Y \to bY \mid aY \mid \varepsilon \quad ?$$

Give a CFG for the language

$$\{w\ \#0^n \mid n \geq 0, |w| = n\}$$

Give a PDA recognizing the language

$$\{0^n 1^n \mid n \geq 0\}$$

Prove that $\{w \in \{0,1\}^* \mid w \text{ is a palindrome with the same number of 0s and 1}s\}$ is not context-free