

BU CS 332 – Theory of Computation

Lecture 4:

- Non-regular languages
- Pumping Lemma

Reading:
Sipser Ch 1.4

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Pumping Lemma (Formal)

Let L be a regular language.

Then there exists a “pumping length” p such that

For every $w \in L$ where $|w| \geq p$,

w can be split into three parts $w = xyz$ where:

1. $|y| > 0$
2. $|xy| \leq p$
3. $xy^iz \in L$ for all $i \geq 0$



Pumping Lemma as a game

1. **YOU** pick the language L to be proved nonregular.
2. **ADVERSARY** picks a possible pumping length p .
3. **YOU** pick w of length at least p .
4. **ADVERSARY** divides w into x, y, z , obeying rules of the Pumping Lemma: $|y| > 0$ and $|xy| \leq p$.
5. **YOU** win by finding $i \geq 0$, for which $xy^i z$ is not in L .

If *regardless* of how the **ADVERSARY** plays this game, you can always win, then L is nonregular

Problem 1:

$$L = \{ww^R \mid w \in \{0,1\}^*\}$$

Show that L is not regular

Problem 2:

$$L = \{0^i 1^j \mid 0 \leq i < j\}$$

Show that L is not regular

Problem 3:

$$L = \{0^i 1^j \mid i > j \geq 0\}$$

Show that L is not regular

Problem 4:

$$L = \{0^i 1^j \mid i \neq j\}$$

Show that L is not regular