Discussion 9

- 1. (Enthusiastic TM) Consider the problem of determining whether a given TM ever (on some input) writes "332" on three adjacent cells of its tape. You may assume that the input alphabet of this TM is {0,1} and the tape alphabet is {0,1,2,...,9}.
 - (a) Formulate this problem as a language ENTHUSIASTIC $_{\mathsf{TM}}.$
 - (b) Show ENTHUSIASTIC $_{\mathsf{TM}}$ is undecidable.

(c) Prove that $\mathrm{ENTHUSIASTIC}_{\mathsf{TM}}$ is Turing-recognizable.

(d) Is $\overline{\mathrm{ENTHUSIASTIC}_{\mathsf{TM}}}$ Turing-recognizable? Prove or disprove.

- 2. (Mapping reductions) Let $\mathsf{CFL}_{\mathsf{TM}} = \{ \langle M \rangle | M \text{ is a TM that recognizes a context-free language} \}$. Prove the following statements about $\mathsf{CFL}_{\mathsf{TM}}$.
 - (a) $\mathsf{CFL}_{\mathsf{TM}}$ is not Turing-recognizable.
 - (b) $\overline{\mathsf{CFL}_{\mathsf{TM}}}$ is not Turing-recognizable (i.e., $\mathsf{CFL}_{\mathsf{TM}}$ is not co-Turing-recognizable).