Discussion 8

Problems

1. (a) Show that $\mathbb{Z}^3 = \{(x, y, z) | \ x, y, z \in \mathbb{Z}\}$ is countable.

   (b) Show that the set of all integer functions $\mathcal{F} = \{f : \mathbb{Z} \to \mathbb{Z}\}$ is uncountable.

2. (**Smooth TM**) Let $\Sigma = \{0, 1, \ldots, 9\}$. A string $w_1 w_2 \ldots w_n$, where $w_1, w_2, \ldots, w_n \in \Sigma$, is *smooth* if, for each position $i \in \{1, \ldots, n-1\}$ in the string, $|w_{i+1} - w_i| \leq 1$. In other words, as we read the string, each subsequent digit is the same as the previous one or differs from it by 1. For example, `56544321210` is smooth, but `576` is not. Observe that ε and 1-digit strings are smooth, because they do not violate the smoothness requirement.

   A TM is *smoothness-obsessed* if it accepts a string if and only if it is smooth. The problem is to determine whether a given TM is a smoothness-obsessed.

   (a) Formulate the given problem as a language L_{smooth}.

   (b) Reduce the halting problem to this language. That is, describe a computable translation $T : \{0,1\}^* \to \{0,1\}^*$ from an instance of the halting problem to an instance of smoothness-obsessed problem L_{smooth}.

   (c) Conclude that this language is undecidable using a proof by contradiction.