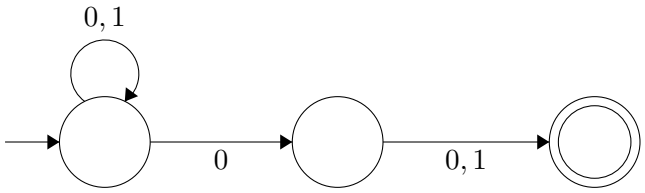
Discussion 2

Problems

1. (**Equivalence of NFAs and DFAs**) Consider the following NFA.



(a) Which language does this NFA recognize? (describe in English)

(b) Build an equivalent DFA using the construction given in the proof that DFAs and NFAs are equivalent.
Note: For this problem, please draw all 8 states, even if a state is unreachable (if any).

2. (**Closure of regular languages**)

   (a) Prove that for any DFA $M = (Q, \Sigma, \delta, q_0, F)$, you can convert it into an NFA N with only one accepting state.

   (b) Given the observation in (a), your TA attempted to simplify the proof of closure of regular languages under concatenation and star, given below. Are these constructions correct? If so, finish the proof; if not, give a counterexample where the construction fails.

   Closure under concatenation: Let L_1, L_2 be two regular languages over the same alphabet Σ, and $N_1 = (Q_1, \Sigma, \delta_1, q_{S,1}, \{q_{A,1}\}), N_2 = (Q_2, \Sigma, \delta_2, q_{S,2}, \{q_{A,2}\})$ be the corresponding NFAs with a single accepting state. Construct an NFA for $L_1 \circ L_2$ by taking N_1, N_2 and merging the accepting state $q_{A,1}$ with the starting state $q_{S,2}$.
   (this is simpler as we no longer use ε-transitions in this construction)

   Closure under star: Let L be a regular language and $N = (Q, \Sigma, \delta, q_S, \{q_A\})$ be the corresponding NFA with a single accepting state. Construct an NFA for L^* by taking N and adding the following ε-transitions: (a) from q_S to q_A; (b) from q_A to q_S.
   (this is simpler as we did not increase the size of NFA in this construction)

3. (**Number of states**) Sipser, 1.69.

   Let $\Sigma = \{0, 1\}$. Let $WW_k = \{ww \mid w \in \Sigma^* \text{ and } w \text{ is of length } k\}$. Show that for each k, no DFA can recognize WW_k with fewer than 2^k states.
   Hints: If a DFA enters different states after reading two different input strings xz and yz with the same suffix z then the DFA must enter different states after reading input strings x and y. (Explain why.) Find 2^k strings on which every DFA recognizing WW_k must enter different states. (Start by finding two such strings.)