Discussion – Wednesday, December 2, 2020

Problems

1. A **clique** in an undirected graph is a subgraph, wherein every two nodes are connected by an edge. A k**-clique** is a clique that contains k nodes. The clique problem is to determine whether a graph contains a clique of a specified size. Let

$$\text{CLIQUE} = \{\langle G, k\rangle |\ G \text{ is an undirected graph with a } k\text{-clique}\}.$$

   (a) Show that CLIQUE is in NP.

   (b) Show that CLIQUE is NP-hard, conclude that it is NP-complete.

2. (**Search vs decision**) In this problem, you will prove that if P=NP, you can factor integers in polynomial time.

   (a) Consider the following language:
   $F = \{\langle a, b, c\rangle \mid a, b, c \text{ are binary integers and } a = pq \text{ for } b \leq p \leq c\}$. Prove that $F \in$ NP.

   (b) If P = NP, the proof you just gave for part (a) implies that $F \in$ P, that is, in polynomial time you can verify whether a number has a factor in a given range. Show that there is a polynomial-time Turing machine that factors any integer.
   Hint: Assume you have a subroutine that does it, and use it repeatedly to *find* a factor of x in polynomial time.