Discussion 11

Problems

1. A *permutation* on a set $\{1, 2, \ldots, n\}$ is a one-to-one, onto function of the form $f : \{1, 2, \ldots, n\} \to \{1, 2, \ldots, n\}$. If f is a permutation, then f^t denotes the composition of f with itself t times. Given permutations f and g and an integer t, you need to determine if $g = f^t$. Formulate this problem as a language and prove that this language is in P.

2. Define a *RAM Turing machine* to be a Turing machine that has *random access memory*. We formalize this as follows: the machine has additional two symbols on its alphabet we denote by `R` and `W` and an additional state we denote by q_{access}. We also assume that the machine has an infinite array A that is initialized to all blanks. Whenever the machine enters q_{access}, if its address tape starts with $\langle i \rangle R$ then the value $A[i]$ is written in the cell next to the `R` symbol. If its tape contains $\langle i \rangle \mathtt{W}\sigma$ where σ is some symbol in the machine's alphabet, then $A[i]$ is set to the value σ.

   Show that if a language L can be decided within time $T(n)$ by a RAM TM, then it is in $\text{TIME}(T(n)^4)$. (Note that extended Church-Turing thesis only tells you that it *should* be in $\text{TIME}\left(T(n)^{O(1)}\right)$.)

   Hint: Try to compress A at any point of time in the execution to size $O(T(n))$, simulate the RAM TM with a 2-tape TM and apply the transformation in the lecture that turns a multi-tape TM with running time T' to a regular TM with running time T'^2.