



2. (**Induction.**) Suppose we are trying to divide a class of  $n$  students into groups of either 4 or 5 students.

(a) Find an error in the following proof that a class with  $n \geq 8$  students can be divided into groups of 4 or 5. That is, identify the first incorrect sentence and explain what went wrong.

*Proof.* The proof is by strong induction. Let  $P(n)$  be the proposition that a class with  $n$  students can be divided into teams of 4 or 5.

**Base case:** We prove that  $P(n)$  is true for  $n = 8, 9$ , and 10 by showing how to break classes of these sizes into groups of 4 or 5 students:

$$8 = 4 + 4;$$

$$9 = 4 + 5;$$

$$10 = 5 + 5.$$

**Induction hypothesis:** Next, we must show that  $P(8), \dots, P(n)$  imply  $P(n+1)$  for all  $n \geq 10$ . That is, we assume that  $P(8), \dots, P(n)$  are all true and show how to divide up a class of  $n+1$  students into groups of 4 or 5. We first form one group of 4 students. Then we can divide the remaining  $n-3$  students into groups of 4 or 5 by the assumption  $P(n-3)$ . This proves  $P(n+1)$ , and so the claim holds by induction.  $\square$

(b) Provide a correct strong induction proof that a class with  $n \geq 12$  students can be divided into groups of 4 or 5.

3. Given a string  $w$  of 0s and 1s, the *flip* of  $w$  is obtained by changing all 0s in  $w$  to 1s and all 1s in  $w$  to 0s. Given a language  $A$ , the flip of  $A$  is the language  $\{w \mid \text{the flip of } w \text{ is in } A\}$ . Prove that if  $L$  is regular then the flip of  $L$  is regular too.