Discussion 1

Course website: https://bu-cs332.github.io PS1 out yesterday, due next Tuesday before lecture... Start early!

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Problems

- 1. (Review of basic logic and set notation.)
 - (a) Negate the following statement: "The first DFA has an even number of states, but the second DFA does not."
 - (b) Negate the following statement: "Every CS major who is awesome is enrolled in CS 332."
 - (c) Write the contrapositive of the following statement: "Your homework assignment is not graded if you have not handed in a signed copy of the Collaboration policy."
 - (d) If $A = \{1, \{2, 3\}, \varepsilon\}$, what is the power set of A?
 - (e) Choose five distinct integers between 0 and 7, inclusive. Show that there exists a pair of them which add up to 7.

- 2. (Induction.) Suppose we are trying to divide a class of n students into groups of either 4 or 5 students.
 - (a) Find an error in the following proof that a class with n ≥ 8 students can be divided into groups of 4 or 5. That is, identify the first incorrect sentence and explain what went wrong.
 Proof. The proof is by strong induction. Let P(n) be the proposition that a class with n students can be divided into teams of 4 or 5.

Base case: We prove that P(n) is true for n = 8, 9, and 10 by showing how to break classes of these sizes into groups of 4 or 5 students:

Induction hypothesis: Next, we must show that P(8), ..., P(n) imply P(n+1) for all $n \ge 10$. That is, we assume that P(8), ..., P(n) are all true and show how to divide up a class of n+1 students into groups of 4 or 5. We first form one group of 4 students. Then we can divide the remaining n-3 students into groups of 4 or 5 by the assumption P(n-3). This proves P(n+1), and so the claim holds by induction.

(b) Provide a correct strong induction proof that a class with $n \ge 12$ students can be divided into groups of 4 or 5.

3. Given a string w of 0s and 1s, the *flip* of w is obtained by changing all 0s in w to 1s and all 1s in w to 0s. Given a language A, the flip of A is the language $\{w \mid \text{the flip of } w \text{ is in } A\}$. Prove that if L is regular then the flip of L is regular too.